`eforensics` Analysis of the Venezuela 2024 Presidential Elections*

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Abstract

I use *mesa* data to estimate `eforensics`-frauds to measure the magnitude of malevolent distortions of electors' intentions—frauds—in the 2024 election for president in Veneuela. The *mesa* data available for me to analyze were collected and published by the leading opposition party and comprise 81.7 percent of the *actas* from the election (Resultados con VZLA 2024). In these data opposition candidate Edmundo González has 7156462 votes compared to 3241461 votes for incumbent Nicolás Maduro, out of 10659128 votes cast for one of ten candidates. Treating the candidate with most votes (González) as the leader for the `eforensics` model—i.e., as the candidate who can benefit from election frauds—estimates show that only two of 24532 mesas in the analysis have `eforensics`-frauds, and the number of `eforensics`-fraudulent votes is scant. The posterior mean total number of `eforensics`-fraudulent votes is 57.9 and the 99.5% credible interval for that total includes zero as a lower bound. There may be no `eforensics`-fraudulent votes among the votes for González at all. The model does not exhibit MCMC posterior multimodality for the mixture probabilities, so there is no evidence of lost votes nor other model misspecification. I show that other recent elections in Venezuela have more `eforensics`-frauds and `eforensics`-fraudulent votes.

1 The 2024 Venezuela President Election

The 2024 election for president in Venezuela prompted extreme controversy due to an attempt by the apparently losing incumbent to declare victory and remain in office (e.g. Schmidt and Brown 2024). Most observers believe that an opposition candidate is the legitimate winner of the election, based in large part on a collection of *actas* that tally the votes at each *mesa* for, as of this writing, about 80 percent of *mesas* (Resultados con VZLA 2024). Both the Associated Press (Cano, Goodman and Kastanis 2024) and the Washington Post (Schmidt, Rich, Herrero and Paúl 2024) report stories that confirm the totals from that collection of *actas*, with the Post in the same story reporting further steps taken to "corroborate the authenticity of tally sheets posted online." These *actas* produce the vote totals for the ten candidates who received votes in the election that are reported in Table 1. Edmundo González Urrutia has the most votes (7156462) and is the apparent winner, while the incumbent Nicolás Maduro has many fewer votes (3241461). Other candidates' vote totals are more than an order of magnitude smaller.

I use `eforensics` (Ferrari, Mebane, McAlister and Wu 2019) to analyze the available *mesa* data for the 2024 election and subsequently for other elections in Venezuela. The `eforensics` model is a finite mixture model where the three distributions that are components of the mixture—the three types of `eforensics`-frauds—are described as "no fraud," "incremental frauds" and "extreme frauds." The model estimates the number of `eforensics`-fraudulent votes for a candidate the analyst chooses before running the model to be the "leader," who is the candidate who can benefit from "election frauds" as these are defined by the model. As I argue in a book I'm completing, the `eforensics`-fraudulent votes are valid but imperfect measures of what I call realized frauds, realized frauds being manifestations of malevolent distortions of electors' intentions that change or can change election outcomes.[1] Bayesian estimation of the model includes estimating unconditional probabilities that each *mesa* has abstention and vote totals generated by each of the three

[1] An elector is anyone who is eligible, or registered, to vote.

Table 1: Venezuela 2024 Election Vote and Elector Totals

Contest	Candidate (Party or Coalition) or Feature	Count
President		
	Edmundo González Urrutia (Independent Unitary Platform)	7156462
	Nicolás Maduro (United Socialist Party of Venezuela)	3241461
	Luis Eduardo Martínez (Democratic Action)	84446
	José Brito (Venezuela First)	21583
	Antonio Ecarri Angola (Pencil Alliance)	49603
	Enrique Márquez (CENTRADOS)	25570
	Benjamn Rausseo (National Democratic Confederation (CONDE))	37408
	Javier Bertucci (EL CAMBIO)	19966
	Claudio Fermn (Solutions for Venezuela)	12321
	Daniel Ceballos (AREPA)	10308
Eligible Voters and Misvotes		
	electors ("RE")	17745239
	votos validos	10659128
	votos nulos	1139

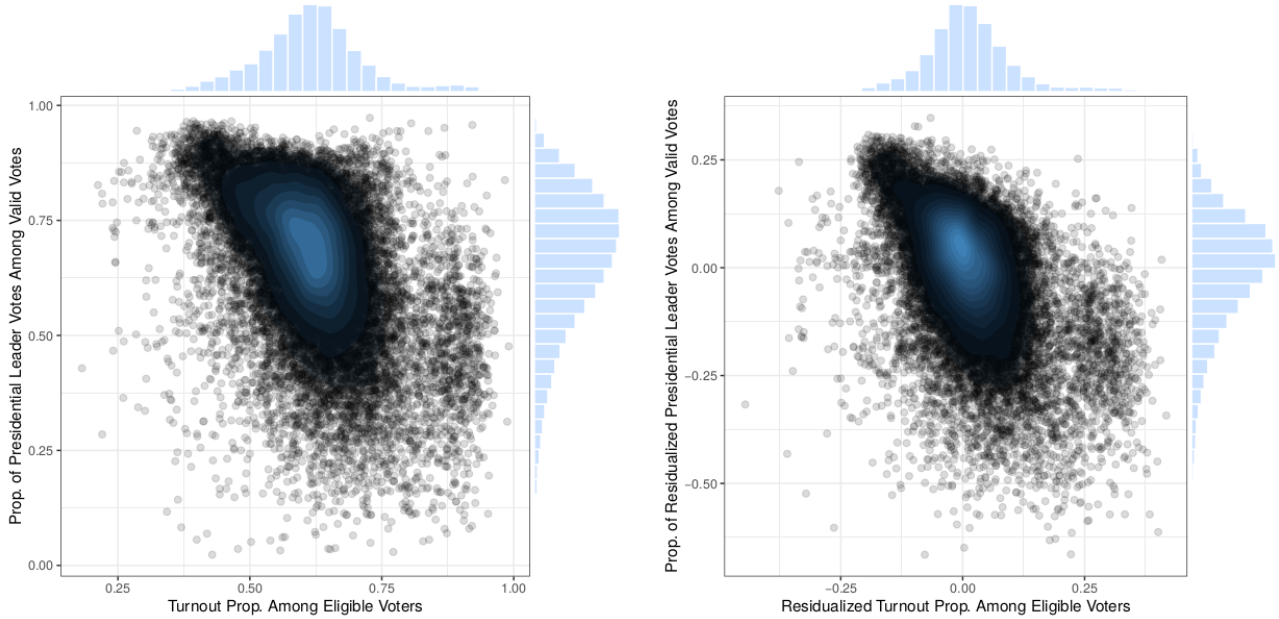
Note: number of voters and vote totals by candidate for the $n = 24532$ *mesas* in Resultados con VZLA (2024) data.

mixture components. These are the mixture probabilities. The estimation procedure also produces parameter estimates that I use to classify each *mesa* as having one of the three types of `eforensics`-frauds and to characterize the number of `eforensics`-fraudulent votes for each *mesa* that has either incremental or extreme `eforensics`-frauds. For a description of the `eforensics` model and estimation procedure and a discussion of model ambiguities and how MCMC posterior multimodality for mixture probabilities relates to model misspecification see Mebane (2023).

For `eforensics` I define the leader to be the candidate with the most votes (González).

Figure 1 shows `eforensics`-plots for *mesa* turnout and leader vote proportions. An `eforensics`-plot shows a scatterplot of the two kinds of proportions with histograms along the margins and a two-dimensional empirical density shown behind the scatterplot's points. For 2024 *mesa* data come from every *estado* except *Embajadas* and *Zonas Inhóspitas*. Figure 1(a) plots the original data while Figure 1(b) plots the data after removing *estado*

Figure 1: `eforensics`-plots: Venezuela 2024 President



(a) original data (b) *estado*-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. $n = 24532$ *mesas*. For `eforensics` estimates see Table 2. Entropy: residualized observed (b), 8.28; Normal simulation, 9.97; efficiency, .9928.

fixed effects. The latter represents the data as they are being treated in the `eforensics` estimates reported in Table 2, because that model specification includes *estado* fixed effects for turnout and vote choice. Overall the original data distribution lacks alarming features: both proportions in Figure 1(a) appear nearly unimodal if skewed. The residualized data have a slightly different appearance, but the distribution is largely unimodal but skewed. Figure 1(b) is mildly clumpy: the efficiency value (.9928) is greater than those for all but nine of the thirty elections listed in Tables 5 and 6 of (Mebane 2023).

Estimates from the `eforensics` model in Table 2 show there are scant `eforensics`-frauds or `eforensics`-fraudulent votes. The model specification includes *estado* fixed effects for turnout and vote choice.[2] For this and all the other models I report in this paper MCMC estimation uses four chains the results from which are combined to

[2]The fixed effects for turnout and vote choice are dummy variable covariates included in x^τ and x^ν in equations (4a) and (4b) in Mebane (2023).

Table 2: Venezuela 2024 President Election `eforensics` Estimates, *Estado* Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.9997	.9994	.99995
	π_2	Incremental Fraud	.000185	1.22e-08	.000465
	π_3	Extreme Fraud	.000114	4.72e-06	.000259
incremental frauds	ρ_{M0}	(Intercept)	$-.948$	-1.05	$-.857$
	ρ_{S0}	(Intercept)	$-.775$	$-.863$	$-.620$
extreme frauds	δ_{M0}	(Intercept)	.00968	$-.0987$	.127
	δ_{S0}	(Intercept)	.0721	$-.0105$	.167

MCMC posterior multimodality diagnostics:
dip test p-values $D(\pi_1) = .69$; $D(\pi_2) = .999$; $D(\pi_3) = 1$.[c]
means difference $M(\pi_1) = .000125$; $M(\pi_2) = 8.29\text{e-}05$; $M(\pi_3) = 5.51\text{e-}05$.[d]

units `eforensics`-fraudulent: (0 incremental, 2 extreme, 24530 not fraudulent)
manufactured votes $F_t = 45.9\ [0.0, 79.6]$[e]
total `eforensics`-fraudulent votes $F_w = 57.9\ [0.0, 99.6]$[e]

Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). *Estado* fixed effects for turnout and vote choice are not shown. $n = 24532$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 17745239$; $\sum_{i=1}^{n} V_i = 10659128$; $\sum_{i=1}^{n} W_i = 7156462$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis (Hartigan and Hartigan 1985) over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

produce the results reported in the table (see Mebane (2023) for details). Symbols in the "Parameter" column of Table 2 correspond to parameters in the formal definition of the model that can be seen in Mebane (2023, 5–8). The very high estimate for the "no frauds" mixture probability, which has a posterior mean and 95% HPD interval of $\pi_1 = .9997$ (.9994, .99995) means that the probabilities that either incremental or extreme frauds occur are very low; indeed, $\pi_2 = .000185$ (1.22e-08, .000465) and $\pi_3 = .000114$ (4.72e-06, .000259). In fact when the classification approach described by Mebane (2023, 8) is applied only two *mesas* of the $n = 24532$ *mesas* in the analysis have `eforensics`-frauds. Both of these `eforensics`-frauds are extreme frauds, which means they are very likely to stem from malevolent distortions of electors' intentions, but the number of `eforensics`-fraudulent votes associated with these *mesas* is vanishingly small.

That is, the total number of `eforensics`-fraudulent votes (posterior mean and 99.5% credible interval) is $F_w = 57.9$ $[0.0, 99.6]$: the posterior mean is extremely small, and the 99.5% credible interval has a lower bound of zero. Such a credible interval means there may be no `eforensics`-fraudulent votes among the votes for González at all.

Mebane (2023) motivates the values reported as MCMC posterior multimodality diagnostics as ways to assess whether the `eforensics` model is misspecified as applied to particular data. In particular, often elections feature lost votes, which are increases in abstentions that asymmetrically benefit either the leader or opposing candidates. The diagnostics reported in Table 2 do not exhibit MCMC posterior multimodality for the mixture probabilities; for example, $D(\pi_2) = .999$ is not significant and $M(\pi_2) = 8.29\text{e-}05$ is not large. So there is no evidence of lost votes nor of other model misspecification.

If nonetheless *estado* fixed effects for frauds magnitudes are added to the specification[3] used to produce Table 2, the results are essentially the same. In this case the "no frauds" mixture probability is $\pi_1 = .9996$ $(.999, .99997)$, only two *mesas* have `eforensics`-frauds and these are incremental, and the total number of `eforensics`-fraudulent votes is $F_w = 147.0$ $[0.0, 315.1]$.[4] The scant *mesas* with `eforensics`-frauds plus the 99.5% credible interval for F_w that has a lower bound of zero again means there may be no `eforensics`-fraudulent votes among the votes for González. A nuance is that with the `eforensics`-frauds being incremental it is relevant that the incremental frauds magnitudes for the *estados* that contain the *mesas* that have `eforensics`-frauds are negative,[5] which means the `eforensics`-frauds, if they exist, are unknown admixtures of malevolent distortions and electors' strategic behaviors. In my book I explain why I say this.

[3]The fixed effects for frauds magnitudes are dummy variable covariates included in x^ι and x^υ in equations (4c) and (4d) in Mebane (2023). These produce *estado*-specific offsets ρ_{Mj}, ρ_{Sj}, δ_{Mj} and δ_{Sj} to the intercepts ρ_{M0}, ρ_{S0}, δ_{M0} and δ_{S0} reported in Table 2.

[4]One *mesa* has `eforensics`-frauds for both specifications. This is *centro* 182501013 *mesa* 1 in *estado* Tachira, *municipio* Mp. Jose M. Vargas, *parroquia* Cm. El Cobre. The number of `eforensics`-fraudulent votes for this *mesa* is $F_{wi} = 43.4$ $[38.9, 47.1]$ for the specification of Table 2 and $F_{wi} = 43.6$ $[39.3, 46.7]$ for the specification that includes the *estado* fixed effects.

[5]For the *estados* that contain the two *mesas* posterior means are, respectively, $\rho_{M0}+\rho_{Mj} = -.860-.0337$ and $\rho_{S0} + \rho_{Sj} = -.776 + .0194$, and $\rho_{M0} + \rho_{Mj} = -.860 - .0469$ and $\rho_{S0} + \rho_{Sj} = -.776 + .0504$.

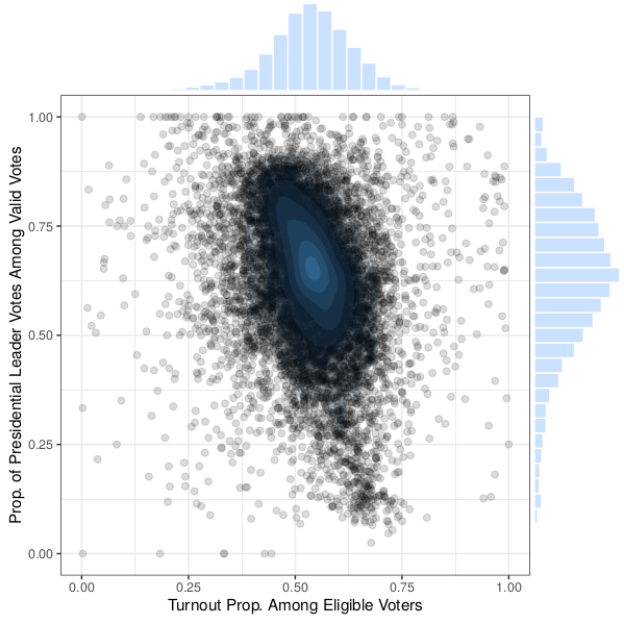
2 Other Elections in Venezuela

To provide some analytical context for the 2024 president `eforensics` estimates, I present `eforensics` results accompanied by some terse discussion for several other elections in Venezuela during 2000–2013. The elections include president and president recall elections and two constitutional referenda.

Figure 2 shows `eforensics`-plots for *mesa* data for the 2000 president and 2004 president recall elections. The leader for 2000 is Chavez, the candidate with the most votes, and the leader for 2004 is No, the ballot alternative with the most votes. For 2000 I have data from every *estado* except *Zonas Inhóspitas*, and for 2004 I have data from every *estado* except *Embajadas* and *Zonas Inhóspitas*. The original data for 2024, in Figure 1(a), does not much resemble the original data for either 2000 (Figure 2(a)) or 2004 (Figure 2(c)). Nor does the *estado*-residualized plot for 2024 (Figure 1(b)) closely resemble either of the residualized plots for 2000 (Figure 2(b)) or 2004 (Figure 2(d)).

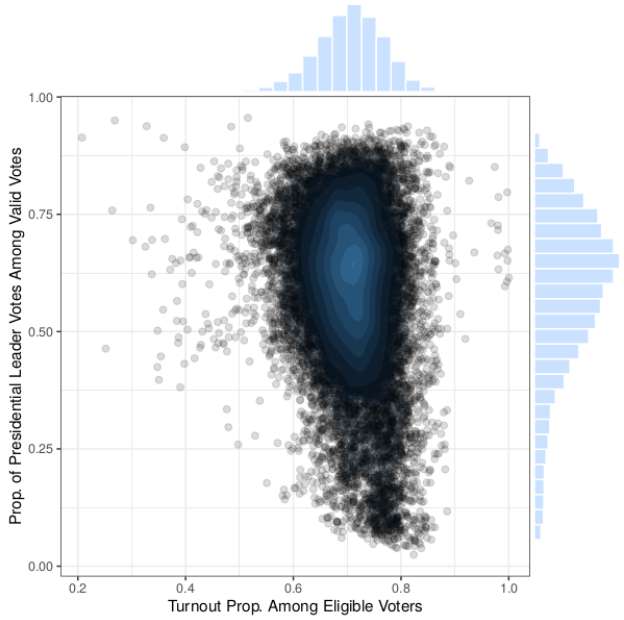
The `eforensics` estimates reported in Table 3 show that while there are not all that many `eforensics`-frauds or `eforensics`-fraudulent votes for 2000, there are more for 2000 than occur for 2024. The model specification includes *estado* fixed effects for turnout and vote choice, and using this specification the MCMC posterior multimodality diagnostics do not exhibit MCMC posterior multimodality for the mixture probabilities; for example, $D(\pi_2) = 1$ is not significant and $M(\pi_2) = .00352$ is not large. The estimate for the "no frauds" mixture probability, $\pi_1 = .977$ $(.973, .982)$, is not as high as the value for 2024. For 2000, $\pi_2 = .0207$ $(.0160, .0249)$ and $\pi_3 = .00199$ $(.00106, .00294)$ are large enough that 42 *mesas* have incremental frauds while 23 *mesas* have extreme frauds. The total number of `eforensics`-fraudulent votes for 2000 is $F_w = 6870.0$ $[5396.7, 8567.5]$, which is clearly positive and much larger than occurs for 2024. Nonetheless F_w is a very small proportion of the 3727631 leader votes.

Figure 2: `eforensics`-plots: Venezuela 2000 President and 2004 President Recall

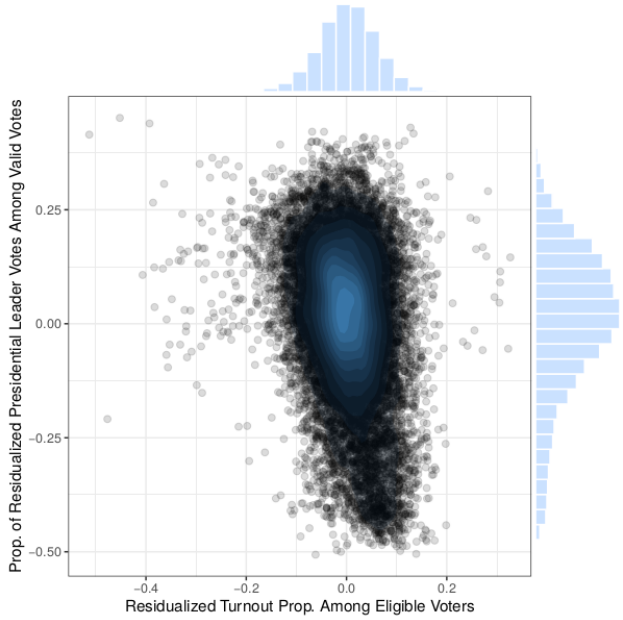


(a) 2000 original data

(b) 2000 *estado*-residualized data



(c) 2004 original data



(d) 2004 *estado*-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. (a,b) $n = 10337$ and (c,d) $n = 19064$ *mesas*. For `eforensics` estimates see Tables 3, 4 and 5. Entropy for 2000: residualized observed (b), 7.73; Normal simulation, 9.20; efficiency, .9976. Entropy for 2004: residualized observed (b), 8.31; Normal simulation, 9.72; efficiency, .9957.

Table 3: Venezuela 2000 President Election `eforensics` Estimates, *Estado* Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.977	.973	.982
	π_2	Incremental Fraud	.0207	.0160	.0249
	π_3	Extreme Fraud	.00199	.00106	.00294
incremental frauds	ρ_{M0}	(Intercept)	$-.441$	$-.577$	$-.307$
	ρ_{S0}	(Intercept)	$-.404$	$-.481$	$-.240$
extreme frauds	δ_{M0}	(Intercept)	$-.0133$	$-.0834$	.0732
	δ_{S0}	(Intercept)	$-.282$	$-.547$	$-.122$

MCMC posterior multimodality diagnostics:
dip test p-values $D(\pi_1) = .997$; $D(\pi_2) = 1$; $D(\pi_3) = 1$.[c]
means difference $M(\pi_1) = .00359$; $M(\pi_2) = .00352$; $M(\pi_3) = .000468$.[d]

units `eforensics`-fraudulent: (42 incremental, 23 extreme, 10272 not fraudulent)

manufactured votes	$F_t = 4932.4\ [3838.8, 6302.3]$[e]
incremental manufactured	$F_t = 2845.7\ [1820.4, 4175.0]$[e]
extreme manufactured	$F_t = 2086.7\ [1752.8, 2292.5]$[e]
total `eforensics`-fraudulent votes	$F_w = 6870.0\ [5396.7, 8567.5]$[e]
incremental total	$F_w = 4047.2\ [2713.2, 5723.5]$[e]
extreme total	$F_w = 2822.8\ [2352.4, 3101.9]$[e]

Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). *Estado* fixed effects for turnout and vote choice are not shown. $n = 10337$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 11620151$; $\sum_{i=1}^{n} V_i = 6243243$; $\sum_{i=1}^{n} W_i = 3727631$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

While the 23 extreme frauds and $F_w = 2822.8\ [2352.4, 3101.9]$ extreme `eforensics`-fraudulent votes for 2000 very likely stem from malevolent distortions, the negative values of the incremental frauds magnitude intercepts (ρ_{M0} and ρ_{S0}) mean that the incremental `eforensics`-frauds and $F_w = 4047.2\ [2713.2, 5723.5]$ incremental `eforensics`-fraudulent votes are unknown admixtures of malevolent distortions and electors' strategic behaviors.

For the 2004 president recall election Table 4 shows while there are more `eforensics`-frauds and `eforensics`-fraudulent votes than occur for 2024, there are fewer than occur for 2000. Now the model specification does not include *estado* fixed effects for

Table 4: Venezuela 2004 President Recall Election `eforensics` Estimates

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.996	.994	.998
	π_2	Incremental Fraud	.00399	.00212	.00595
	π_3	Extreme Fraud	.0000952	3.52e-08	.000298
turnout	β_0	(Intercept)	.892	.875	.916
vote choice	γ_0	(Intercept)	.353	.327	.380
incremental frauds	ρ_{M0}	(Intercept)	$-.367$	-1.01	$-.0392$
	ρ_{S0}	(Intercept)	$-.252$	$-.663$	$-.0309$
extreme frauds	δ_{M0}	(Intercept)	.0789	$-.124$	.190
	δ_{S0}	(Intercept)	.0195	$-.132$	.183

MCMC posterior multimodality diagnostics:
dip test p-values $D(\pi_1) = 1$; $D(\pi_2) = 1$; $D(\pi_3) = 1$.[c]
means difference $M(\pi_1) = .00191$; $M(\pi_2) = .00175$; $M(\pi_3) = .000163$.[d]

units `eforensics`-fraudulent: (22 incremental, 0 extreme, 19042 not fraudulent)
manufactured votes $F_t = 1419.6\ [280.6, 2008.9]$[e]
total fraudulent votes $F_w = 2284.3\ [567.7, 3135.4]$[e]

Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). $n = 19064$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 12090216$; $\sum_{i=1}^{n} V_i = 8505867$; $\sum_{i=1}^{n} W_i = 4920465$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

turnout and vote choice. Using this specification the MCMC posterior multimodality diagnostics do not exhibit MCMC posterior multimodality for the mixture probabilities; for example, $D(\pi_2) = 1$ is not significant and $M(\pi_2) = .00175$ is not large. The estimate for the "no frauds" mixture probability, $\pi_1 = .996$ $(.994, .998)$, is just slightly smaller that is the value for 2024. For 2004, $\pi_2 = .00399$ $(.00212, .00595)$ and $\pi_3 = .0000952$ (3.52e-08, .000298) are so small that only 22 *mesas* have `eforensics`-frauds, all incremental. The negative values of the incremental frauds magnitude intercepts (ρ_{M0} and ρ_{S0}) mean that the total number of `eforensics`-fraudulent votes for 2004, $F_w = 2284.3\ [567.7, 3135.4]$, which is clearly positive and a bit larger than occurs for 2024, is an unknown admixture of malevolent distortions and electors' strategic behaviors.

Table 5 shows that for 2004 adding *estado* fixed effects for turnout, vote choice and

Table 5: Venezuela 2004 President Recall Election `eforensics` Estimates, *Estado* Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.997	.994	.9997
	π_2	Incremental Fraud	.00227	7.87e-07	.00597
	π_3	Extreme Fraud	.000328	6.86e-05	.000656
MCMC posterior multimodality diagnostics:					
dip test p-values	$D(\pi_1) = .502$; $D(\pi_2) = .798$; $D(\pi_3) = .873$.[c]				
means difference	$M(\pi_1) = .0034$; $M(\pi_2) = .00362$; $M(\pi_3) = .00022$.[d]				
units `eforensics`-fraudulent: (0 incremental, 5 extreme, 19059 not fraudulent)					
manufactured votes	$F_t = 416.7\ [349.5, 515.0]$[e]				
total `eforensics`-fraudulent votes	$F_w = 768.2\ [662.7, 942.4]$[e]				

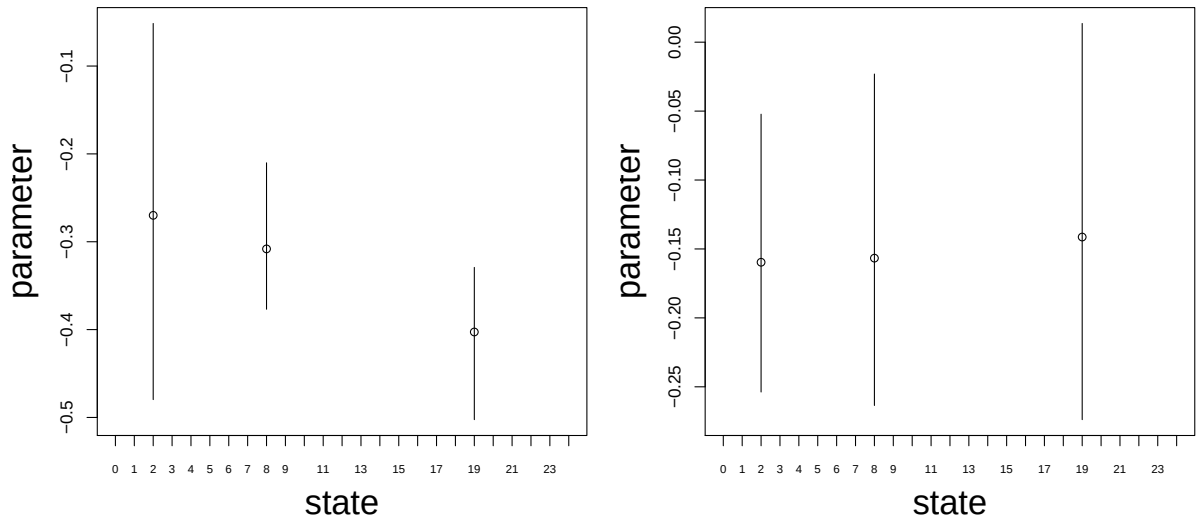
Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). *Estado* fixed effects for turnout, vote choice and `eforensics`-frauds magnitudes are not shown (see Figure 3 for active fraud magnitude fixed effects). $n = 19064$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 12090216$; $\sum_{i=1}^{n} V_i = 8505867$; $\sum_{i=1}^{n} W_i = 4920465$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

frauds magnitudes produces roughly the same results, with a nuanced change that may be important. Using this specification the MCMC posterior multimodality diagnostics do not exhibit MCMC posterior multimodality for the mixture probabilities; for example, $D(\pi_2) = .798$ is not significant and $M(\pi_2) = .00362$ is not large. The estimate for the "no frauds" mixture probability, $\pi_1 = .997$ $(.994, .9997)$, is just slightly smaller that is the value for 2024. Including the fixed effects, for 2004 $\pi_2 = .00227$ $(7.87\text{e-}07, .00597)$ and $\pi_3 = .000328$ $(6.86\text{e-}05, .000656)$ are so small that only 5 *mesas* have `eforensics`-frauds, all extreme. With the fixed effects included the total number of `eforensics`-fraudulent votes for 2004 is $F_w = 768.2\ [662.7, 942.4]$, which is clearly positive and a bit larger than occurs for 2024.

With *estado* fixed effects included for frauds magnitudes, it is convenient to use displays like the one in Figure 3 to show how for 2004 the frauds magnitudes vary across *estados*.

Figure 3: Venezuela 2004 President Recall: `eforensics`-frauds Magnitude Fixed Effect Parameters

(a) extreme manufactured: δ_{Mj} (b) extreme stolen: δ_{Sj}



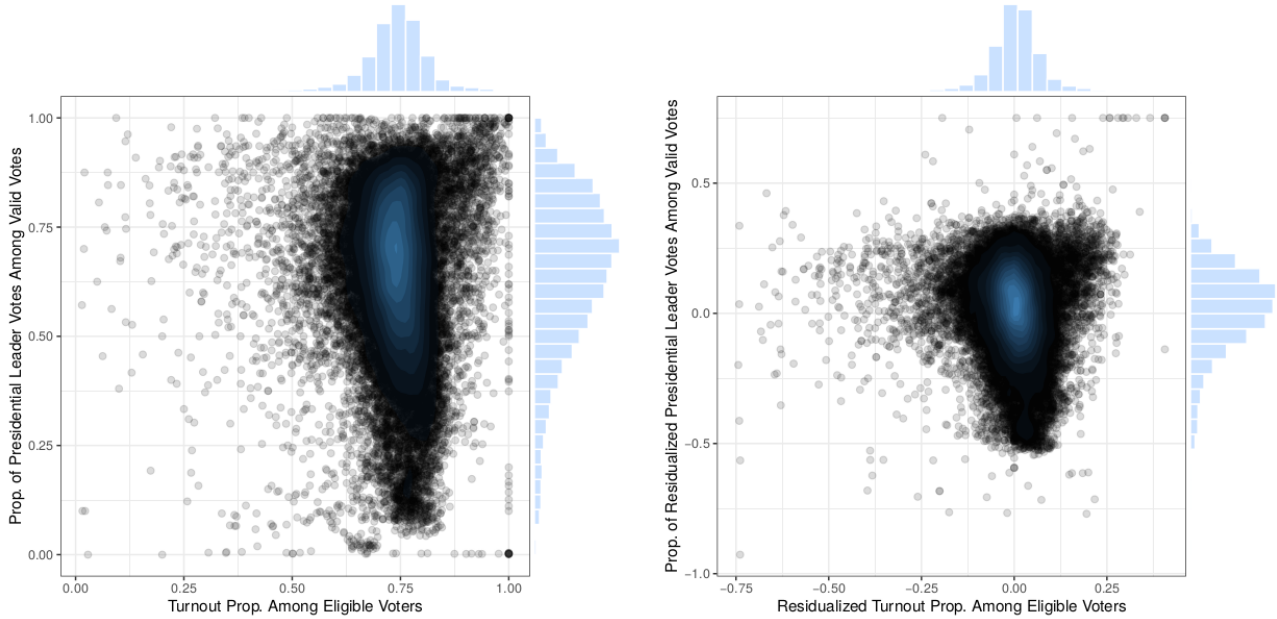
Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{Mj}, ρ_{Sj}, δ_{Mj}, δ_{Sj}) parameters in the `eforensics` model reported in Table 5. The states with `eforensics`-frauds are: (extreme) 2 Guárico, 8 Portuguesa, 19 Aragua.

The figure displays what I call active fixed effects: I say a fixed effect is active if it is associated with a *mesa* that has the corresponding type of `eforensics`-frauds. For each *estado* j that has an active fixed effect, Figure 3 shows $\delta_{M0} + \delta_{Mj}$ and $\delta_{S0} + \delta_{Sj}$.[6] The five *mesas* that have extreme frauds occur in three *estados*, so Figure 3 displays only three active fixed effects each for manufactured ($\delta_{M0} + \delta_{Mj}$) and stolen ($\delta_{S0} + \delta_{Sj}$) frauds magnitudes. The manufactured fixed effects (Figure 3(a)) appear to vary across *estados* more than do the stolen fixed effects (Figure 3(b)).

Because extreme frauds are very likely to stem from malevolent distortions of electors' intentions, perhaps the results of the 2004 specification that includes all the *estado* fixed

[6] A caveat is that for all fixed effects except any displayed in position zero, which corresponds to the intercept, I simply add the posterior mean of the intercept to the posterior mean of the fixed effect's coefficient and to the limits of its 95% HPD interval, without adjusting for how these intervals should change to represent the full variation of the combined fixed effects. Variation due to uncertainty about the intercept and the dependence between the intercept and each fixed effect coefficient is not included. So pending implementation of such corrected credible intervals, the displays in Figure 3 and similar figures in this paper should be viewed merely as informally illustrative.

Figure 4: `eforensics`-plots: Venezuela 2006 President



(a) original data (b) *estado*-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. $n = 33031$ *mesas*. For `eforensics` estimates see Table 6. Entropy: residualized observed (b), 8.88; Normal simulation, 10.2; efficiency, .9932.

effects (Table 5 and Figure 3) better match the findings of papers like Delfino and Salas (2011) than do the results of the specification that omits all fixed effects (Table 4), even though the MCMC posterior multimodality dignostics do not motivate adding any fixed effects. The results in Table 4 seem more in line with the conclusions reached by Carter Center (2005).

Figure 4 shows `eforensics`-plots for *mesa* data for the 2006 president election. The leader for 2006 is Chavez, the candidate with the most votes. For 2006 I have data from every *estado*. The original data for 2006, in Figure 4(a), somewhat resemble the original data for 2000 (Figure 2(a)), and the *estado*-residualized data are somewhat similar to the 2000 data as well (Figures 4(b) and 2(b)).

The `eforensics` estimates reported in Table 6 show that the 2006 election differs greatly from both the 2000 and 2024 elections. The model specification includes *estado*

Table 6: Venezuela 2006 President Election `eforensics` Estimates, *Estado* Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.731	.493	.979
	π_2	Incremental Fraud	.257	.00851	.494
	π_3	Extreme Fraud	.0117	.00816	.0142

MCMC posterior multimodality diagnostics:

dip test p-values	$D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 0$.[c]
means difference	$M(\pi_1) = .483$; $M(\pi_2) = .483$; $M(\pi_3) = .00423$.[d]

units `eforensics`-fraudulent: (777 incremental, 408 extreme, 31846 not fraudulent)

manufactured votes	$F_t = 23013.3$ $[17037.8, 27567.8]$[e]
incremental manufactured	$F_t = 8502.7$ $[2929.1, 11315.1]$[e]
extreme manufactured	$F_t = 14510.5$ $[11403.4, 16324.4]$[e]
total `eforensics`-fraudulent votes	$F_w = 55932.3$ $[33372.0, 78247.4]$[e]
incremental total	$F_w = 27397.9$ $[6216.1, 45367.2]$[e]
extreme total	$F_w = 28534.4$ $[21128.3, 33133.4]$[e]

Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). *Estado* fixed effects for turnout, vote choice and `eforensics`-frauds magnitudes are not shown (see Figure 5 for active fraud magnitude fixed effects). $n = 33031$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 11925880$; $\sum_{i=1}^{n} V_i = 11764337$; $\sum_{i=1}^{n} W_i = 7386666$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

fixed effects for turnout, vote choice and frauds magnitudes, but even using this specification the MCMC posterior multimodality diagnostics exhibit MCMC posterior multimodality for the mixture probabilities; for example, $D(\pi_2) = 0$ is significant and $M(\pi_2) = .483$ is large. The estimate for the "no frauds" mixture probability, $\pi_1 = .731$ $(.493, .979)$, is low and $\pi_2 = .257$ $(.00851, .494)$ and $\pi_3 = .0117$ $(.00816, .0142)$ are large. Given $\pi_2 = .257$ it is notable that the number of *mesas* that have incremental frauds is only 777 among $n = 33031$ total *mesas*: $777/33031 = .0235 \ll .257 = \pi_2$. The reason for this is same reason the 95% HPD intervals for π_1 and π_2 are so wide: the MCMC posterior multimodality is nearly as extreme as it can be.

Table 7 reports estimates for 2006 for the mixture probabilities based separately on each of the four chains used to produce the results in Table 6, showing that the estimates

Table 7: Venezuela 2006 President Election `eforensics` Estimates, Chain-specific Mixture Probabilities

Type	Parameter	Covariate	Mean	lo[a]	up[b]
chain 1	π_1	No Fraud	.977	.974	.981
	π_2	Incremental Fraud	.0101	.00596	.0135
	π_3	Extreme Fraud	.0128	.0114	.0140
chain 2	π_1	No Fraud	.494	.493	.495
	π_2	Incremental Fraud	.493	.493	.494
	π_3	Extreme Fraud	.0124	.0113	.0137
chain 3	π_1	No Fraud	.494	.493	.495
	π_2	Incremental Fraud	.493	.492	.494
	π_3	Extreme Fraud	.0129	.0116	.0142
chain 4	π_1	No Fraud	.960	.956	.963
	π_2	Incremental Fraud	.0317	.0284	.0350
	π_3	Extreme Fraud	.00872	.00768	.00977

Note: chain-specific mixture probability estimates for the model specifications reported in Table 6 (posterior means and credible intervals). [a] 95% HPD lower bound. [b] 95% HPD upper bound.

for π_1 and π_2 vary greatly across chains. Chains 2 and 3 have posterior modes for which $\pi_1 \approx \pi_2$, while for the other two chains $\pi_1 \gg \pi_2$. The latter two chains also differ from each other with respect to π_1 and π_2. The overall posterior mean of $\pi_2 = .257$ corresponds to none of the chain-specific values. Notice that estimates for π_3 mostly agree across chains. A *mesas*'s classification as having incremental or extreme frauds requires consensus among a plurality of the chains (see Mebane (2023, 8)), and such agreements do not occur for incremental frauds as frequently as $\pi_2 = .257$ may suggest.

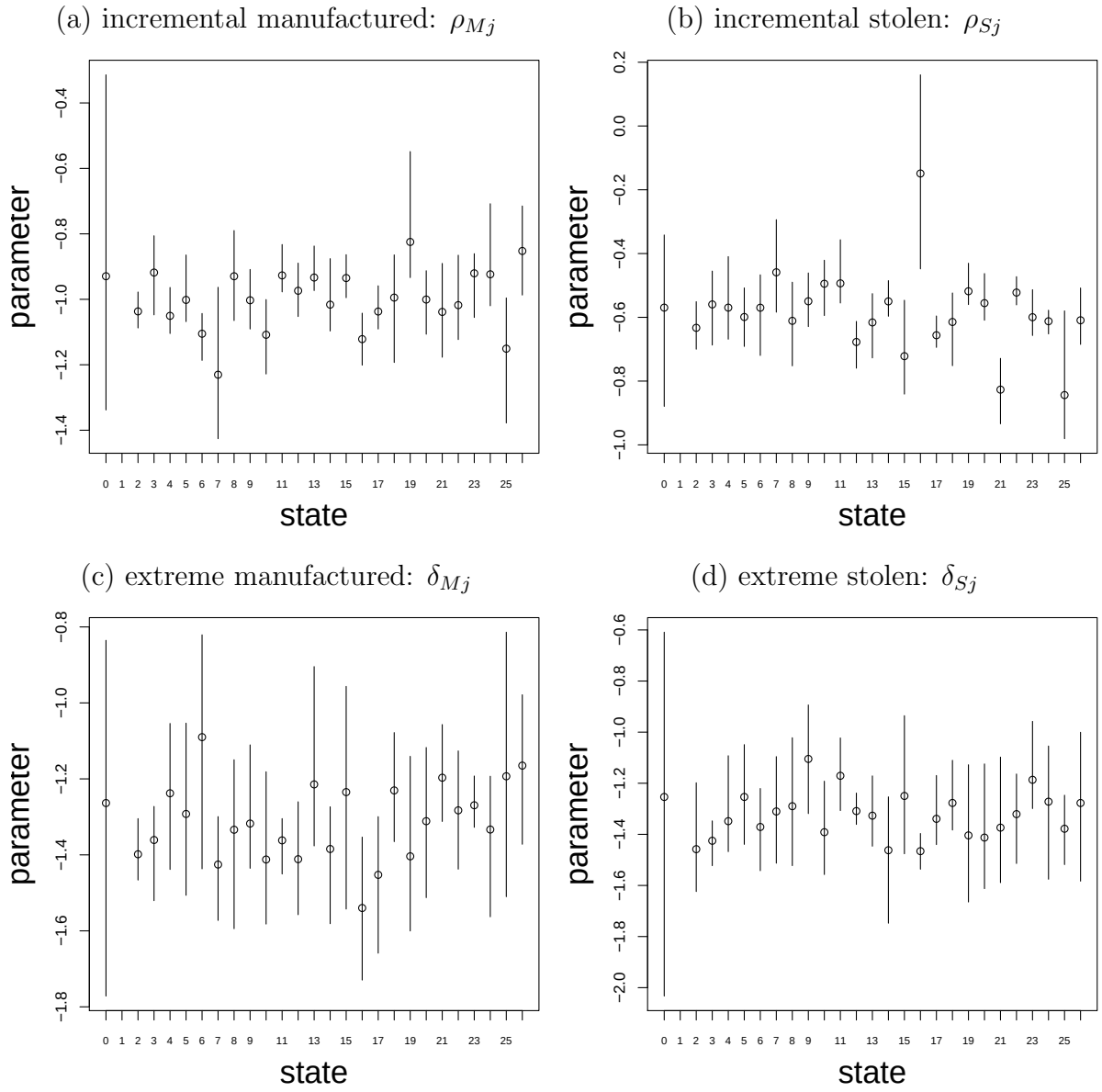
Such posterior multimodality explains why the 99.5% credible interval for the incremental `eforensics`-fraudulent votes ($F_w = 27397.9$ $[6216.1, 45367.2]$) is so wide—the upper bound is more than seven times as large as the lower bound. This large uncertainty carries into the estimate for the overall `eforensics`-fraudulent votes total ($F_w = 55932.3$ $[33372.0, 78247.4]$). Perhaps the count of 408 *mesas* with extreme frauds is right, as well as the total of extreme `eforensics`-fraudulent votes ($F_w = 28534.4$ $[21128.3, 33133.4]$), but the large MCMC posterior multimodality makes the

estimates of incremental frauds unreliable, likely even more unreliable than the HPD intervals for π_1 and π_2 reported in Table 6 may suggest.

The primary reason for such mixture probability MCMC posterior multimodality, as suggested by Mebane (2023), is lost votes. Either electors who would have supported the leader or the opposition asymmetrically decline to vote, or votes cast for the leader or the opposition are asymmetrically not counted—perhaps the votes are spoiled—hence effectively become abstentions. Decisions not to vote may be voluntary, even strategic, or they may result from intimidations. Votes willfully spoiled by a third party or intimidations are types of malevolent distortions of electors' intentions. Whether such malevolent distortions manifest as part of F_w is unclear.

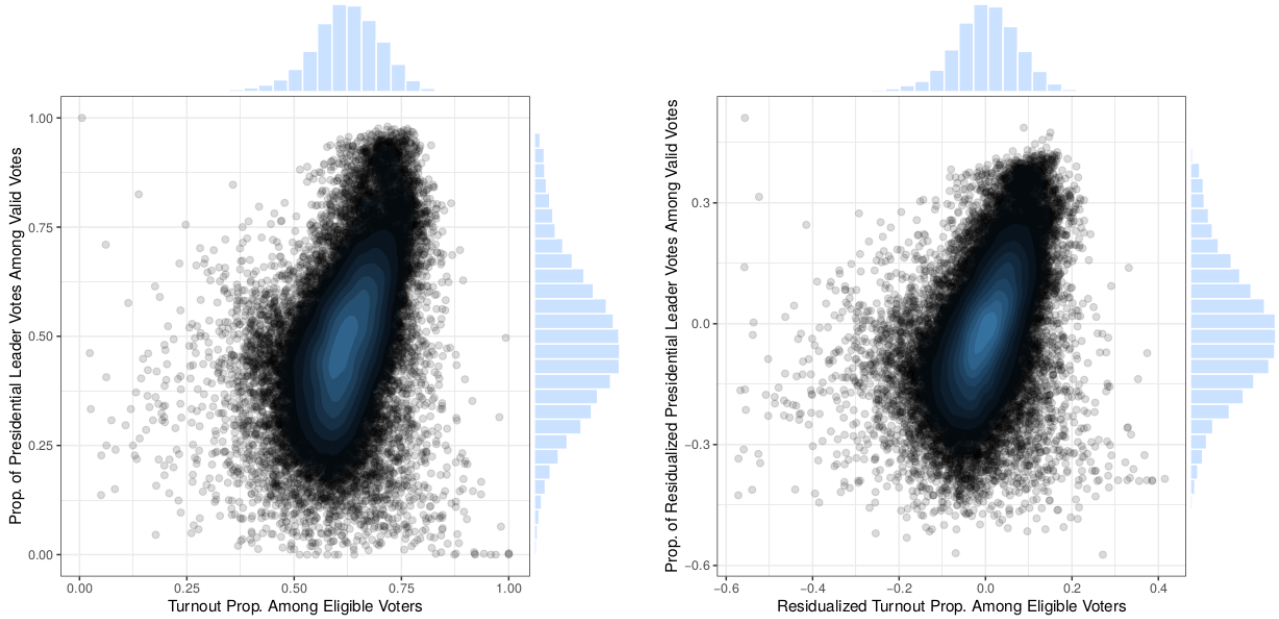
So while Figure 5 shows active frauds magnitudes fixed effects for 2006 for every *estado*, it is not clear at least whether the incremental frauds magnitudes fixed effects are accurate. That the posterior means of the incremental frauds magnitudes fixed effects are all negative is some evidence that the incremental `eforensics`-fraudulent votes may be admixtures of malevolent distortions and electors' strategic behavior, but the situation is not as clear as when extreme MCMC posterior multimodality does not occur.

Figure 5: Venezuela 2006 President: `eforensics`-frauds Magnitude Fixed Effect Parameters



Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{Mj}, ρ_{Sj}, δ_{Mj}, δ_{Sj}) parameters in the `eforensics` model reported in Table 6. The states with `eforensics`-frauds are: (incremental) 0 Amazonas, 2 Anzoátegui, 3 Apure, 4 Aragua, 5 Barinas, 6 Bolívar, 7 Carabobo, 8 Cojedes, 9 Delta Amacuro, 10 Dtto. Capital, 11 Embajada, 12 Falcón, 13 Guárico, 14 Lara, 15 Mérida, 16 Miranda, 17 Monagas, 18 Nueva Esparta, 19 Portuguesa, 20 Sucre, 21 Táchira, 22 Trujillo, 23 Vargas, 24 Yaracuy, 25 Zulia, 26 Zona Inhóspitas; (extreme) same.

Figure 6: `eforensics`-plots: Venezuela 2007 Constitutional Referendum



(a) original data (b) *estado*-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. $n = 29072$ *mesas*. For `eforensics` estimates see Table 8. Entropy: residualized observed (b), 9.00; Normal simulation, 10.1; efficiency, .9952.

Figure 6 shows `eforensics`-plots for *mesa* data for the 2007 constitutional referendum (Proposal A). The leader for 2007 is No, the ballot alternative with the most votes. For 2007 I have data for every *estado* except *Embajadas* and *Zonas Inhóspitas*. The original data for 2007, in Figure 6(a), somewhat resemble the original data for 2000 (Figure 2(a)), but the *estado*-residualized data are not all that similar to the data for 2000 (Figures 6(b) and 2(b)).

The `eforensics` estimates reported in Table 8 show that the 2007 election differs greatly from the 2004 election. The model specification includes *estado* fixed effects for turnout and vote choice, and with these fixed effects the MCMC posterior multimodality diagnostics do not exhibit MCMC posterior multimodality for the mixture probabilities; for example, $D(\pi_2) = .999$ is not significant and $M(\pi_2) = .00314$ is not large. The estimate for the "no frauds" mixture probability, $\pi_1 = .857$ $(.852, .861)$, is low and at least

Table 8: Venezuela 2007 Constitutional Referendum Election `eforensics` Estimates, *Estado* Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.857	.852	.861
	π_2	Incremental Fraud	.143	.139	.148
	π_3	Extreme Fraud	3.77e-05	9.86e-09	.000119
incremental frauds	ρ_{M0}	(Intercept)	$-.438$	$-.467$	$-.423$
	ρ_{S0}	(Intercept)	.382	.274	.474
extreme frauds	δ_{M0}	(Intercept)	$-.0957$	$-.176$	$-.0135$
	δ_{S0}	(Intercept)	.0179	$-.0446$	.102

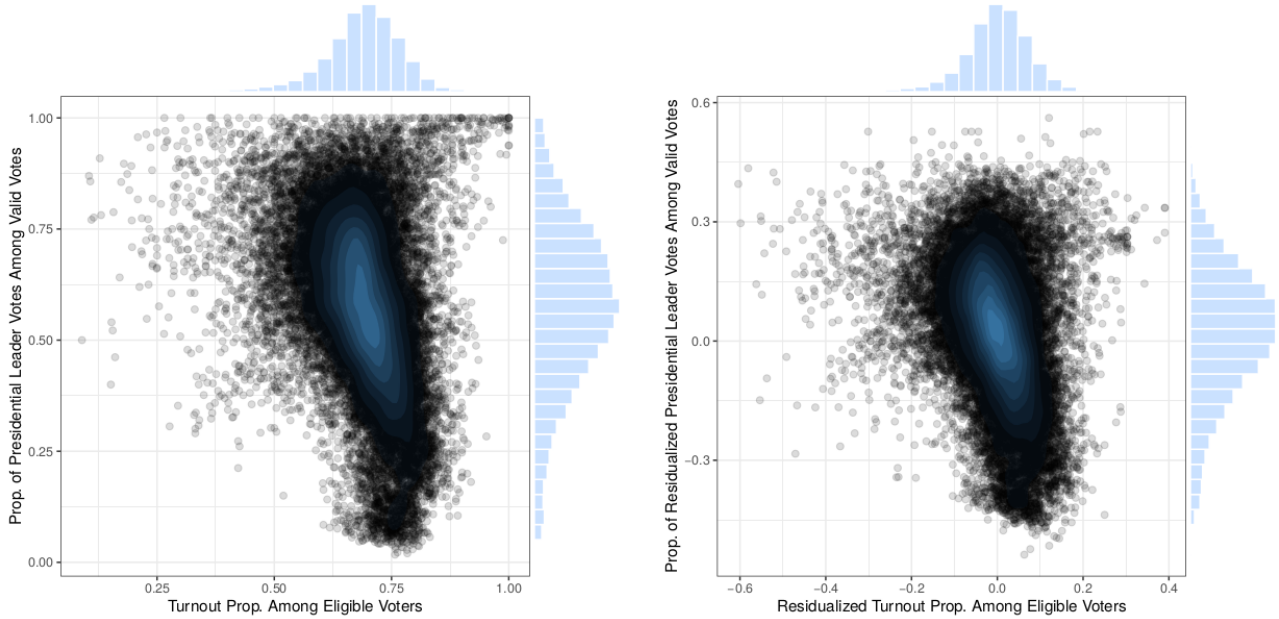
MCMC posterior multimodality diagnostics:
dip test p-values $D(\pi_1) = .998$; $D(\pi_2) = .999$; $D(\pi_3) = 1$.[c]
means difference $M(\pi_1) = .00314$; $M(\pi_2) = .00314$; $M(\pi_3) = 4.96\text{e-}06$.[d]

units `eforensics`-fraudulent: (3705 incremental, 0 extreme, 25367 not fraudulent)
manufactured votes $F_t = 173815.7\ [167325.0, 178900.7]$[e]
total `eforensics`-fraudulent votes $F_w = 408611.6\ [383550.2, 422691.7]$[e]

Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). *Estado* fixed effects for turnout and vote choice are not shown. $n = 29072$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 14299478$; $\sum_{i=1}^{n} V_i = 8883746$; $\sum_{i=1}^{n} W_i = 4504354$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

$\pi_2 = .143$ $(.139, .148)$ is large; $\pi_3 = 3.77\text{e-}05$ $(9.86\text{e-}09, .000119)$. Note that the number of *mesas* that have incremental frauds, 3705, as a proportion of $n = 29072$ *mesas* is only slightly less than π_2: $3705/29072 = .127$; this contrasts with the situation for the 2006 president election. The number of *mesas* that have `eforensics`-frauds for 2007 greatly exceeds the number for 2004, as does the number of `eforensics`-fraudulent votes. For 2007 the total number of `eforensics`-fraudulent votes is $F_w = 408611.6\ [383550.2, 422691.7]$, most of which are stolen: there are $F_t = 173815.7\ [167325.0, 178900.7]$ manufactured votes, and $F_w - F_t = 408611.6 - 173815.7 = 234795.9$ stolen votes. For 2007 the incremental frauds magnitudes intercept is negative for manufactured votes (ρ_{M0}) but positive for the stolen votes (ρ_{S0}). $\rho_{S0} > 0$ means there is little basis for interpreting the incremental stolen votes as anything but entirely results of malevolent distortions of electors' intentions.

Figure 7: `eforensics`-plots: Venezuela 2009 Constitutional Referendum



(a) original data (b) *estado*-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. $n = 31853$ *mesas*. For `eforensics` estimates see Table 9. Entropy: residualized observed (b), 8.67; Normal simulation, 10.2; efficiency, .9952.

$F_w = 408611.6$ or even $F_w - F_t = 234795.9$ `eforensics`-fraudulent votes is a substantial proportion of leader votes: $408611.6/4504354 = .09$.

Figure 7 shows `eforensics`-plots for *mesa* data for the 2009 constitutional referendum. The leader for 2009 is Yes, the alternative with the most votes. For 2009 I have data for every *estado* except *Embajadas* and *Zonas Inhóspitas*. The original data for 2009, in Figure 7(a) somewhat resemble the original data for 2006 (Figure 4(a)), as do the *estado*-residualized data (Figure 7(b) and 4(b)).

The `eforensics` estimates reported for the 2009 election in Table 9 show that the 2009 election is remarkably similar to the 2024 election except with many more extreme frauds. The model specification for 2009 includes *estado* fixed effects for turnout and vote choice, and with these fixed effects the MCMC posterior multimodality diagnostics do not exhibit MCMC posterior multimodality for the mixture probabilities; for example, $D(\pi_2) = .963$ is

Table 9: Venezuela 2009 Constitutional Referendum `eforensics` Estimates, *Estado* Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.997	.996	.997
	π_2	Incremental Fraud	.000148	3.07e-07	.000394
	π_3	Extreme Fraud	.00318	.00251	.00382
incremental frauds	ρ_{M0}	(Intercept)	$-.644$	$-.775$	$-.511$
	ρ_{S0}	(Intercept)	$-.824$	$-.936$	$-.707$
extreme frauds	δ_{M0}	(Intercept)	$-.134$	$-.199$	$-.0667$
	δ_{S0}	(Intercept)	.0293	$-.0885$	.101

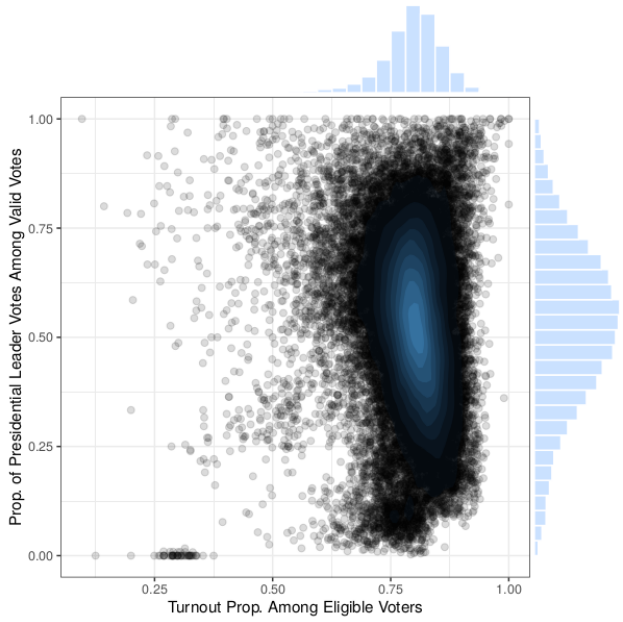
MCMC posterior multimodality diagnostics:
dip test p-values $D(\pi_1) = 1$; $D(\pi_2) = .963$; $D(\pi_3) = 1$.[c]
means difference $M(\pi_1) = .00038$; $M(\pi_2) = 9.02\text{e-}05$; $M(\pi_3) = .000294$.[d]

units `eforensics`-fraudulent: (0 incremental, 100 extreme, 31753 not fraudulent)
manufactured votes $F_t = 5885.2$ $[5642.0, 6361.3]$[e]
total `eforensics`-fraudulent votes $F_w = 9983.4$ $[9511.6, 10623.3]$[e]

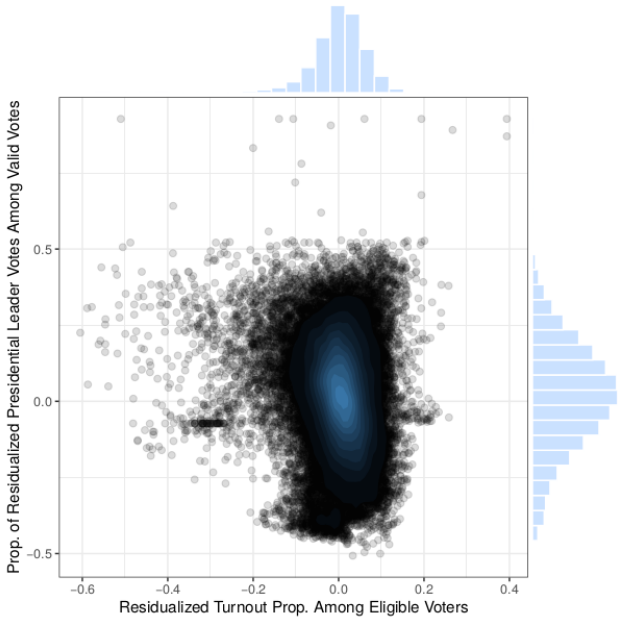
Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). *Estado* fixed effects for turnout and vote choice are not shown. $n = 31853$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 15454792$; $\sum_{i=1}^{n} V_i = 10657385$; $\sum_{i=1}^{n} W_i = 5866607$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

not significant and $M(\pi_2) = 9.02\text{e-}05$ is not large. The estimate for the "no frauds" mixture probability, $\pi_1 = .996$ $(.996, .997)$, is low and both $\pi_2 = .000148$ $(3.07\text{e-}07, .000394)$ and $\pi_3 = .00318$ $(.00251, .00382)$ are small. No *mesas* have incremental frauds, but π_3 is large enough for 100 *mesas* to have extreme `eforensics`-frauds. For 2009 the total number of `eforensics`-fraudulent votes is $F_w = 9983.4$ $[9511.6, 10623.3]$. Being extreme frauds these `eforensics`-fraudulent votes are very likely to be the result of malevolent distortions of electors' intentions.

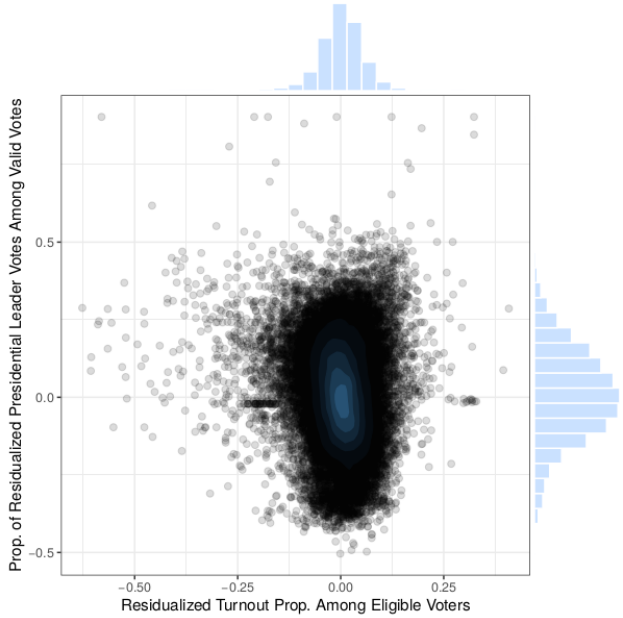
Figure 8: `eforensics`-plots: Venezuela 2013 President



(a) 2013 original data



(b) 2013 *estado*-residualized data



(c) 2013 *municipio*-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. $n = 39298$ *mesas*. For `eforensics` estimates see Table 10. Entropy, *estado*: residualized observed (b), 9.00; Normal simulation, 10.3; efficiency, .9927. Entropy, *municipio*: residualized observed (c), 8.91; Normal simulation, 10.3; efficiency, .9919.

Table 10: Venezuela 2013 President Election `eforensics` Estimates, *Estado* Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.868	.498	.996
	π_2	Incremental Fraud	.129	.000160	.498
	π_3	Extreme Fraud	.00361	.00153	.00508
MCMC posterior multimodality diagnostics:					
dip test p-values	$D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 0$.[c]				
means difference	$M(\pi_1) = .496$; $M(\pi_2) = .496$; $M(\pi_3) = .0028$.[d]				
units `eforensics`-fraudulent: (0 incremental, 130 extreme, 39168 not fraudulent)					
manufactured votes	$F_t = 3524.9$ $[2097.5, 4549.6]$[e]				
total `eforensics`-fraudulent votes	$F_w = 10390.8$ $[5450.1, 13705.7]$[e]				

Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). *Estado* fixed effects for turnout, vote choice and `eforensics`-frauds magnitudes are not shown (see Figure 9 for active fraud magnitude fixed effects). $n = 39298$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 18894164$; $\sum_{i=1}^{n} V_i = 14987727$; $\sum_{i=1}^{n} W_i = 7586459$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

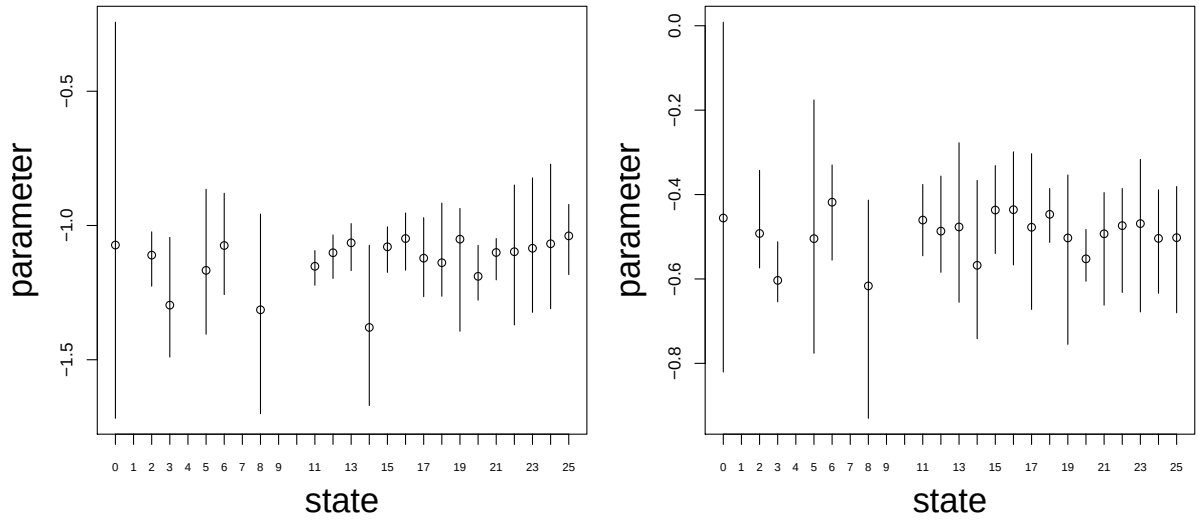
Figure 8 shows `eforensics`-plots for *mesa* data for the 2013 president election. The leader for 2013 is Maduro, the candidate with the most votes. For 2013 I have data for every *estado*. The figure shows the original data plus data residualized both for *estado* fixed effects and for municipio fixed effects.[7] The two residualized plots differ from one another to some extent, and the municipio-residualized scatterplot is very slightly more clumpy according to the efficiency values (.9919 versus .9927).

The reason for the set of fixed effects defined using an administrative level lower than the *estado* is that as Table 10 reports `eforensics` estimates using a specification that includes *estado* fixed effects for turnout, vote choice and frauds magnitudes feature MCMC posterior multimodality like that that occurs for the 2006 election. For example, $D(\pi_2) = 0$ is significant and $M(\pi_2) = .496$ is large. There are 130 *mesas* with `eforensics`-frauds, and all of these are extreme. Figure 9 displays the active frauds magnitudes fixed effects. Using

[7]For the municipio fixed effects all municipios in an *estado* that have fewer than 50 *mesas* are combined into a "small" artificial municipio in the *estado*.

Figure 9: Venezuela 2013 President: `eforensics`-frauds Magnitude Fixed Effect Parameters

(a) extreme manufactured: δ_{Mj} (b) extreme stolen: δ_{Sj}



Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{Mj}, ρ_{Sj}, δ_{Mj}, δ_{Sj}) parameters in the `eforensics` model reported in Table 10 The states with `eforensics`-frauds are: (extreme) . 0 Dtto. Capital, 2 Guárico, 3 Lara, 5 Miranda, 6 Monagas, 8 Portuguesa, 11 Trujillo, 12 Anzoátegui, 13 Yaracuy, 14 Zulia, 15 Amazonas, 16 Delta Amacuro, 17 Vargas, 18 Apure, 19 Aragua, 20 Barinas, 21 Bolívar, 22 Carabobo, 23 Cojedes, 24 Falcón, 25 Embajadas.

instead *municipio* fixed effects for turnout, vote choice and frauds magnitudes (Table 11) produces both *mesas* that have incremental frauds (75) and *mesas* that have extreme frauds (191), but MCMC posterior multimodality persists: still $D(\pi_2) = 0$ and $M(\pi_2) = .496$. Figure 10 displays the active frauds magnitudes fixed effects. In Figures 10(a,b) a symptom of what is going on is the wide HPD interval for both incremental frauds magnitudes intercepts.

Table 12 reports that for 2013, similar to what happens for 2006 (cf. Table 7), estimates for π_1 and π_2 vary greatly across chains. For either specification of the fixed effects, for one chain $\pi_1 \approx \pi_2$, while for at least two other chains $\pi_1 \gg \pi_2$. The overall posterior means of $\pi_2 = .129$ with *estado* fixed effects and $\pi_2 = .232$ with *municipio* fixed effects correspond to none of the chain-specific values. As for 2006, the large MCMC posterior multimodality for

Table 11: Venezuela 2013 President Election `eforensics` Estimates, *Municipio* Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.763	.498	.995
	π_2	Incremental Fraud	.232	3.77e-07	.497
	π_3	Extreme Fraud	.00495	.00394	.00611

MCMC posterior multimodality diagnostics:
dip test p-values $D(\pi_1) = 0$; $D(\pi_2) = 0$; $D(\pi_3) = 1$.[c]
means difference $M(\pi_1) = .495$; $M(\pi_2) = .496$; $M(\pi_3) = .00116$.[d]

units `eforensics`-fraudulent: (75 incremental, 191 extreme, 39032 not fraudulent)

manufactured votes	$F_t = 7530.3\ [6134.3, 11071.4]$[e]
incremental manufactured	$F_t = 1763.2\ [698.7, 3848.2]$[e]
extreme manufactured	$F_t = 5767.1\ [5061.6, 7367.3]$[e]
total `eforensics`-fraudulent votes	$F_w = 19536.1\ [18461.4, 20845.9]$[e]
incremental total	$F_w = 3904.8\ [2470.1, 5382.4]$[e]
extreme total	$F_w = 15631.2\ [14601.2, 16874.7]$[e]

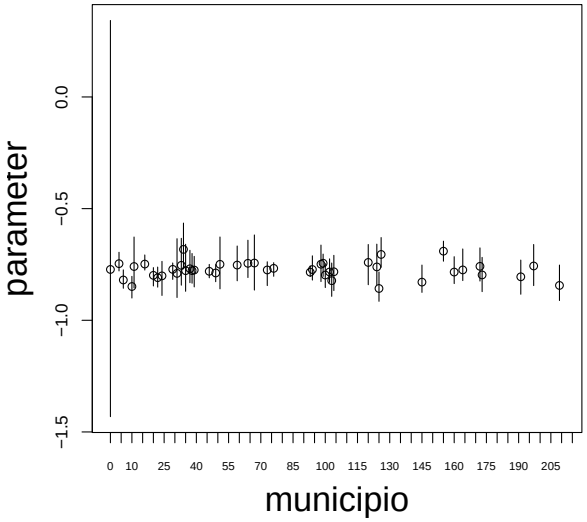
Note: selected `eforensics` model parameter estimates (posterior means and credible intervals). *Municipio* fixed effects for turnout, vote choice and `eforensics`-frauds magnitudes are not shown (see Figure 10 for active fraud magnitude fixed effects). $n = 39298$ *mesa* units. Electors, votes cast and votes for the leader: $\sum_{i=1}^{n} N_i = 18894164$; $\sum_{i=1}^{n} V_i = 14987727$; $\sum_{i=1}^{n} W_i = 7586459$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [c] dip test for unimodality null hypothesis over all MCMC chains. [d] difference between largest and smallest chain-specific posterior means. [e] posterior mean [99.5% credible interval].

2013 makes the estimates of incremental frauds unreliable.

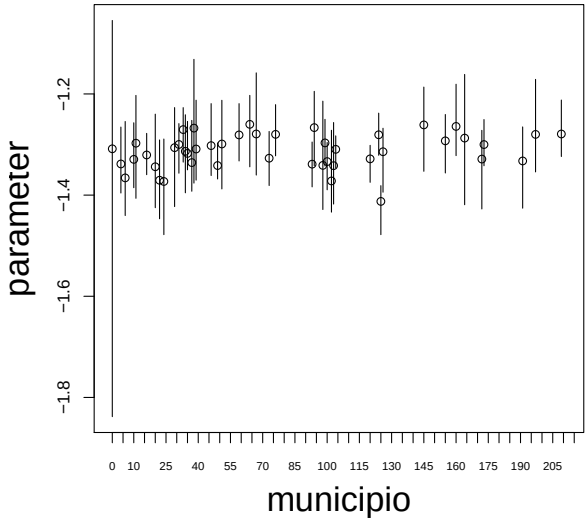
For 2013 as for 2006 the primary reason for such mixture probability MCMC posterior multimodality is lost votes. As for 2006, there may be asymmetric decisions not to vote or asymmetrically spoiled votes. Asymmetric decisions not to vote may be voluntary or they may result from intimidations. Willfully spoiled votes or intimidations are types of malevolent distortions of electors' intentions. Whether such malevolent distortions manifest as part of F_w is unclear.

Figure 10: Venezuela 2013 President: `eforensics`-frauds Magnitude Fixed Effect Parameters

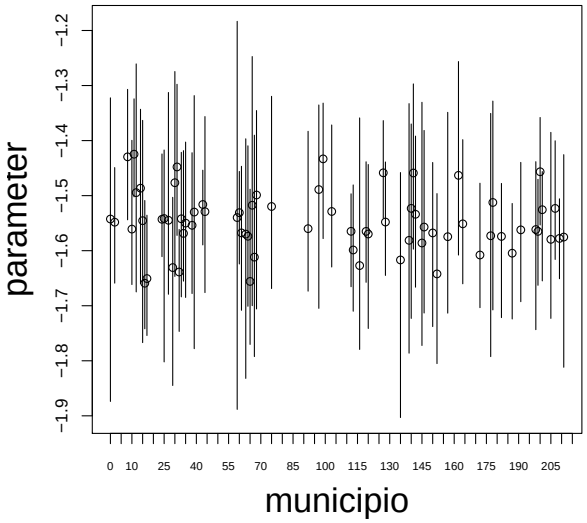
(a) incremental manufactured: ρ_{Mj}



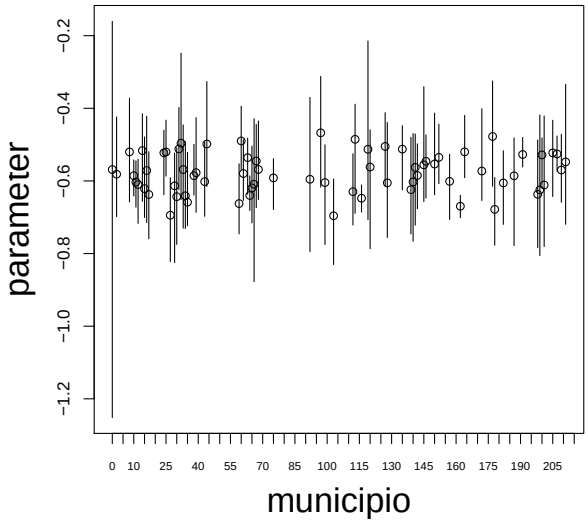
(b) incremental stolen: ρ_{Sj}



(c) extreme manufactured: δ_{Mj}



(d) extreme stolen: δ_{Sj}



Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{Mj}, ρ_{Sj}, δ_{Mj}, δ_{Sj}) parameters in the `eforensics` model reported in Table 11.

Table 12: Venezuela 2013 President Election `eforensics` Estimates, Chain-specific Mixture Probabilities

(a) specification with *Estado* fixed effects (Table 10)

Type	Parameter	Covariate	Mean	lo[a]	up[b]
chain 1	π_1	No Fraud	.994	.991	.996
	π_2	Incremental Fraud	.00180	8.30e-05	.00471
	π_3	Extreme Fraud	.00370	.00314	.00432
chain 2	π_1	No Fraud	.994	.993	.996
	π_2	Incremental Fraud	.000919	8.44e-06	.00240
	π_3	Extreme Fraud	.00468	.00401	.00536
chain 3	π_1	No Fraud	.498	.498	.4998
	π_2	Incremental Fraud	.497	.496	.498
	π_3	Extreme Fraud	.00419	.00352	.00486
chain 4	π_1	No Fraud	.984	.978	.989
	π_2	Incremental Fraud	.0144	.00895	.0208
	π_3	Extreme Fraud	.00188	.146	.00236

(b) specification with *Municipio* fixed effects (Table 11)

Type	Parameter	Covariate	Mean	lo[a]	up[b]
chain 1	π_1	No Fraud	.586	.579	.595
	π_2	Incremental Fraud	.408	.401	.415
	π_3	Extreme Fraud	.00566	.00490	.00651
chain 2	π_1	No Fraud	.499	.497	.501
	π_2	Incremental Fraud	.497	.494	.498
	π_3	Extreme Fraud	.00450	.00385	.00526
chain 3	π_1	No Fraud	.994	.993	.995
	π_2	Incremental Fraud	.000595	3.77e-07	.00162
	π_3	Extreme Fraud	.00498	.00425	.00562
chain 4	π_1	No Fraud	.972	.943	.992
	π_2	Incremental Fraud	.0232	.00316	.0524
	π_3	Extreme Fraud	.00468	.00395	.00533

Note: chain-specific mixture probability estimates for the model specifications reported in Tables 10 and 11 (posterior means and credible intervals). [a] 95% HPD lower bound. [b] 95% HPD upper bound.

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